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TECHNOLOGY****A HAND-EYE CALIBRATION METHOD FOR 6 DOF ROBOT****Liu Jie\*, He Jia-wei, Ping Xue-liang, Qi Fei**

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**ABSTRACT**

For robotic hand-eye calibration, the most important procedure is to solve the hand-eye rotation matrix R and the hand-eye translation matrix T. An effective method is proposed for solving them in this paper, which combines the advantage of the high precision of traditional calibration method with active visual method. The parameters for solving the hand-eye rotation matrix R are found by making the robot perform twice orthogonal translational motion and photographing the calibration plate in each position separately, and the parameters for solving the hand-eye translation matrix T are found by moving the robot to the origin of the calibration plate. The Independently developed Ping-Fang six DOF robot is used as the experiment object to calculate the hand-eye relation matrix of the proposed method. The conducted experiment demonstrate that the accuracy of the proposed method is higher, and it is easier to implement because there is no need to solve the complicated traditional equation.

**KEYWORDS:** Hand-eye calibration, Robot, Translation motion, End-effector.

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**INTRODUCTION**

In the field of industrial robot visual application, there are two kinds of camera installation, Eye-to-Hand and Eye-in-Hand [1-2]. The Eye-to-Hand way of installation may result in some corner and image cover phenomenon, while the Eye-in-Hand way of installation will not appear these problems, because the camera is fixed at the end of the robot moving along with end of the actuator, also it can real-time display the camera image within current view. The Eye-in-Hand vision system is mainly refers to the hand-eye system. When robots perform a task, it is necessary to obtain the position of their work-piece relative to robot base coordinate system, and the key is to determine the relationship between the camera coordinate system and the robot end actuators coordinate system, namely the robot hand-eye calibration [3].

Calibration techniques include two large pieces of content: the camera imaging geometry model and calibration method [4-5]. Pinhole model is commonly used imaging model, its principle is to simplify the projection of the object into pinhole imaging, which is simple and easy to use. Camera calibration methods are divided into three categories: the traditional calibration method, self-calibration method and active vision calibration method [6]. These methods have different advantages and disadvantages. The traditional calibration method must rely on board, the calculation is complicated, but its precision is high and applicable to all kinds of camera imaging model. Self-calibration method does not need calibration board, but need to establish the corresponding relationship between image points, which is generally solved by using absolute quadratic curve or surface, so its robustness is not high and precision is low. Active vision calibration method requires that the camera can move according to specific requirements and have linear solution with strong robustness. In terms of hand-eye calibration, some scholars do related research at home and abroad [7]. Tsai [8] and Shiu [9] proposed to control robot-end executor motion at least two times, in three different directions, camera photograph obtained the size of the calibration plate and solved the hand-eye relation X by establishing hand-eye constraint equation  $AX = XB$ . Later, a number of researchers studied the solution to this equation and introduced a variety of solutions such as our elements and matrix direct product, etc. In 1996, Ma [10] proposed active visual method which is the cornerstone of the next active visual hand-eye calibration method, this method does not need calibration board, use the theory of the pole and establish system of equations to solve hand-eye relation. Zhan-yi Hu [11] and others made a comprehensive analysis on the application of active vision hand-eye calibration.

This paper proposes a hybrid calibration method by combining the traditional calibration method and active vision calibration method, mainly using the camera movement and outside access to get the final hand-eye parameters. The calibration process need move the terminal actuator and obtain an image point location, which is simple and easy to implement. This algorithm avoids the solution of equations  $AX = XB$ , algorithm process is concise and easy to understand, and help the robot positioning artifacts.

### CALIBRATION OF ROBOT HAND-EYE RELATIONSHIP

#### Hand-eye system mathematical model

Hand-eye calibration is the process of computing the conversion relationship  $R, T$  between the camera coordinate system and the robot end executor coordinate system [12],  $R$  is rotation matrix,  $T$  is translation matrix. The robot hand-eye relation diagram is shown in figure 1.

Establish the robot base coordinate system, called  $C_b$ . The end actuator coordinate system is called  $C_e$ , The camera coordinate system is called  $C_c$ , The world coordinate system is called  $C_{obj}$ , The relationship between  $C_c$  and  $C_e$  is  $R, T$ . The camera is fixed on the robot-end, thus hand-eye relation  $R$  and  $T$  is fixed, does not change with movement of the robot. There is a point  $P$  in the field of view, coordinate in the end- actuator coordinate and camera coordinate is  ${}^eP$  and  ${}^cP$  respectively. For the hand-eye relationship:

$${}^cP = R {}^eP + T \quad (1)$$

${}^cP$  can be jointly determined by camera internal and  $P$

pixel coordinate.  ${}^eP$  can be obtained through type(1) , then according to the robot controller for the end-actuator position relative to base coordinate system, the position of point  $P$  under base coordinate system can be obtained by moving the robot ,called  ${}^bP$ .

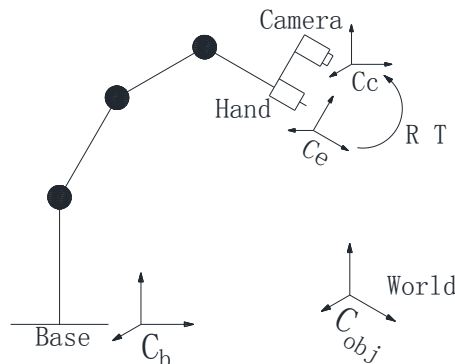


Figure 1 Robot hand-eye relation

#### Calibration of the rotation matrix R in hand-eye relationship

In this paper, control the robot-end executor to carry out two flat movement to calibrate R. For example, in view of point P,  ${}^bP$  represents the coordinate in the robot base frame,  ${}^eP$  represents the coordinate in the end executor frame,  ${}^cP$  represents the coordinate in the camera frame. According to the space coordinate system transformation relations, we can see that:

$${}^bP = {}^bR {}^eP + {}^bT \quad (2)$$

$${}^eP = {}^eR {}^cP + {}^eT \quad (3)$$

${}^eR$  in equation (3) is the inverse of hand-eye rotation matrix R.

Control the robot-end executor to carry out a pure translation motion, moved by the A position to B position. Set the end executor translation vector for  $ka$ ,  $a$  is a unit vector. Point P is described under  $C_e$  after translation as  ${}^eP'$ , point P is described under  $C_c$  after translation as  ${}^cP'$ .

$${}^eP' = {}^eP + ka \quad (4)$$

By the equation (3), we can get that:

$${}^e\mathbf{P}' = {}^e\mathbf{R} {}^c\mathbf{P}' + {}^e\mathbf{T} \quad (5)$$

Take the equation (3) into equation (4), we can get that:

$${}^e\mathbf{P}' = {}^e\mathbf{R} {}^c\mathbf{P} + {}^e\mathbf{T} + \mathbf{ka} \quad (6)$$

Incorporate the equation (5) and (6), we can get that:

$${}^e\mathbf{R} ({}^c\mathbf{P}' - {}^c\mathbf{P}) = \mathbf{ka} \quad (7)$$

The unit vector  $\mathbf{a}$  can be obtained by controlling the robot movement.  ${}^c\mathbf{P}' - {}^c\mathbf{P}$  represents the camera movement. Set  ${}^c\mathbf{P}' - {}^c\mathbf{P} = \mathbf{kb}$ ,  $\mathbf{b}$  for the camera translation unit vector. This paper presents a new method to determine the camera translation vector  $\mathbf{b}$ . Keep the calibration plate actual position unchanged, the camera takes a picture in position A and B respectively to calibrate external reference by utilizing Zhang Zhengyou calibration tool. The one to three lines of the fourth column of external reference matrix actually can be understood as the description of the origin of the calibration plate in the camera coordinate system, the origin of the calibration plate is a fixed point. Therefore, the result of the one to three lines of the fourth column of external reference matrix in B position minus the one to three lines of the fourth column of external reference matrix in A position and  ${}^c\mathbf{P}' - {}^c\mathbf{P}$  is the same, namely  $\mathbf{kb}$ , then through the unitization, a unit vector  $\mathbf{b}$  can be obtained. Therefore, the equation (7) evolved into

$${}^e\mathbf{R} \mathbf{b} = \mathbf{a} \quad (8)$$

Control the robot-end effector carry out a pure translation motion again, moving from B position to C position. The mobile direction is perpendicular to the last direction, therefore, can be obtained again

$${}^e\mathbf{R} \mathbf{b}' = \mathbf{a}' \quad (9)$$

Due to the two vertical moving direction, so the following equation is established.

$${}^e\mathbf{R} (\mathbf{b} \mathbf{b}' \mathbf{b} \mathbf{b}') = (\mathbf{a} \mathbf{a}' \mathbf{a} \mathbf{a}') \quad (10)$$

namely  ${}^e\mathbf{R} \mathbf{B} = \mathbf{A}$ , and  $\mathbf{A} = (\mathbf{a} \mathbf{a}' \mathbf{a} \mathbf{a}')$ ,  $\mathbf{B} = (\mathbf{b} \mathbf{b}' \mathbf{b} \mathbf{b}')$ .

Eventually, the hand-eye rotation matrix is available.

$$\mathbf{R} = \mathbf{B}\mathbf{A}^{-1} \quad (11)$$

Calibration of the translation matrix  $\mathbf{T}$  in hand-eye relationship

This article put forward by controlling the end actuator of the robot movement to calibrate the hand-eye translational matrix  $\mathbf{T}$  at a time. By the equation (1) to be seen

$$\mathbf{T} = {}^c\mathbf{P} - \mathbf{R} {}^e\mathbf{P} \quad (12)$$

$\mathbf{R}$  can be obtained from 2.2 algorithm. Point  $P$  is described under  $C_c$  as  ${}^c\mathbf{P}$ . In this paper, set the calibration board origin as the point  $P$ . Then  ${}^c\mathbf{P}$  represents the one to three lines of the fourth column of external reference matrix of the camera in this position. Through a photograph, then the external reference can get with Zhang Zhengyou Matlab calibration toolbox. According to the space coordinate system transformation relations, to be seen

$${}^e\mathbf{P} = {}^e\mathbf{R} {}^b\mathbf{P} + {}^e\mathbf{T} \quad (13)$$

${}^e\mathbf{R}$  and  ${}^e\mathbf{T}$  can be read by robot controller.  ${}^e\mathbf{R}$  and  ${}^e\mathbf{T}$  can be obtained through the inverse solution. Control the robot end executor move to point  $P$ , at this time, point  $P$  in the base coordinate system described as the  ${}^b\mathbf{P}$  can be read by the robot controller. Take  ${}^b\mathbf{P}$ ,  ${}^e\mathbf{R}$  and  ${}^e\mathbf{T}$  the three parameters into equation (13) can solve  ${}^e\mathbf{P}$ .

Take  ${}^e\mathbf{P}$ ,  ${}^c\mathbf{P}$  and  $\mathbf{R}$  the three parameters into equation (12) can solve the translation matrix  $\mathbf{T}$  in hand-eye relationship.

The summary of the hand-eye calibration process

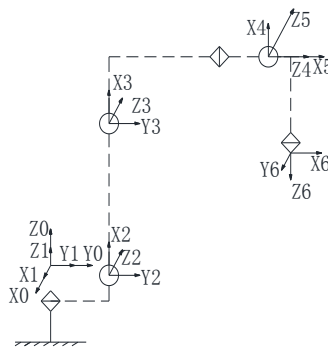
To sum up, hand-eye calibration steps are as follows:

- 1) Carry out the robot-end actuator translational moving from the position A to position B, and set the moving vector for  $ka$ ,  $a$  is a 3 x 1 unit vector, which can be obtained by the robot controller. Set the camera direction for  $kb$ ,  $b$  is a 3 x 1 unit vector, which can be obtained from the result of part of external reference matrix in B position minus part of external reference matrix in A position.
- 2) Choose a direction perpendicular to the direction of the first time to carry out a translation, translate the end-actuator from B to C position, so  $ka', kb'$  is available. According to schmidt orthogonalization procedure, solve rotation matrix  $R$  by equation (11).
- 3) Consider the origin of world coordinate namely the calibration board origin as point P.  ${}^cP$  can be obtained from external reference. Control the robot end actuator moves to point P, so  ${}^bP$  is available. Bring  ${}^bP$  into equation (13) and gain  ${}^cP$ . Bring  ${}^cP$ ,  ${}^cP$  and  $R$  into equation (12), then translation matrix T can be obtained.
- 4) At this point, hand-eye calibration is complete.

## CALIBRATION EXPERIMENT AND ERROR ANALYSIS

The experimental system configuration

In this paper, we use six degrees of freedom serial industrial robot, the second generation Ping-fang, as the research object, which is independent developed and its mathematical model is shown in figure 2, whose link coordinate system is established by D-H method.



**Figure 2 Robot mathematical model**

Choose industrial camera DFK 23GV024, small and easy to install, whose maximum resolution is 752x480, Choose computar industry lens M0814 - MP2, focal length is 8 mm, deformation rate is lower than 1%. The experimental calibration template is black and white checkerboard. There are 9x7 grids in total. Each grid size is 30x30mm.

The implementation process of this calibration algorithm is based on Matlab programming platform. The actual site overall structure as shown in figure 3. Camera fixed at the end of the robot with transfer structure. Based on the left camera as experimental object, to solve the hand-eye relation between the left camera and the end actuator.



**Figure 3 Calibration on site**

The experimental process and result

Using the camera's SDK to write a program which can realize the function of grasping and saving images. Consider the initial robot zero position as A position. Take a calibration plate image, use Zhang Zhengyou Matlab calibration toolbox to get external reference, and save the parameters as  $W_1$ . External calibration diagram is shown in figure 4.

$$W_1 = \begin{bmatrix} 0.959 & 0.256 & -0.121 & 52.747 \\ 0.263 & -0.964 & -0.047 & -111.534 \\ -0.105 & -0.076 & -0.992 & 12540218 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$



Figure 4 External calibration

Using the robot control software to control the robot do translational motion, vector  $a = [0 \ 1 \ 0]^T$ . Keep the calibration plate position unchanged, as the same, get external reference in B position and save as  $W_2$ . Control the robot do translational motion again, vector  $a' = [1 \ 0 \ 0]^T$ . As the same, get external reference in C position and save as  $W_3$ .

$$W_2 = \begin{bmatrix} -0.918 & 0.378 & -0.123 & 76.969 \\ 0.386 & 0.922 & -0.043 & -83.361 \\ -0.097 & -0.086 & -0.992 & 1251.093 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

$$W_3 = \begin{bmatrix} -0.921 & 0.372 & -0.118 & 110.904 \\ 0.379 & 0.924 & -0.041 & -125.892 \\ 0.093 & -0.083 & -0.992 & 1247.376 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

b is obtained by the calculation of  $W_2$  and  $W_1$ .  $b'$  is obtained by the calculation of  $W_3$  and  $W_2$ . So

$$A = [a \ a' \ axa'] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (17)$$

$$B = [b \ b' \ bxb'] = \begin{bmatrix} 0.650 & 0.622 & -0.117 \\ 0.756 & -0.780 & -0.008 \\ -0.084 & -0.068 & -0.977 \end{bmatrix} \quad (18)$$

Bring  $A$  and  $B$  in equation (11),  $R$  can be obtained through calculation and orthogonalization.

$$R = \begin{bmatrix} 0.622 & 0.650 & 0.117 \\ -0.780 & 0.756 & 0.008 \\ -0.068 & -0.084 & 0.977 \end{bmatrix} \quad (19)$$

When robot is located in A position,  ${}^eR$  and  ${}^eT$  are obtained by inversing robot controller value. Control the

robot to move the end executor to the origin of calibration plate. According to the robot control software, record the current position under the base coordinate system, that is  ${}^bP$ .

$${}^eR = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (20)$$

$${}^eT = \begin{bmatrix} -800.14 \\ 0 \\ 476 \end{bmatrix} \quad (21)$$

$${}^bP = \begin{bmatrix} 77.94 \\ 653.58 \\ -675.23 \end{bmatrix} \quad (22)$$

Take  ${}^bP$ ,  ${}^eR$  and  ${}^eT$  these parameters into equation (13) and solve  ${}^eP$ .  ${}^eP$  is obtained by  $W_3$ . Take  ${}^eP$ ,  ${}^eP$  and  $R$  into equation (12) and solve the translation matrix  $T$  in hand-eye relationship.

$${}^eP = \begin{bmatrix} 52.747 \\ -111.534 \\ 1254.218 \end{bmatrix} \quad (23)$$

$$T = \begin{bmatrix} -41.216 \\ -293.790 \\ 126.250 \end{bmatrix} \quad (24)$$

Finally, hand-eye relation matrix  $X$  can be obtained.

$$X = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.622 & 0.650 & 0.117 & -41.216 \\ -0.780 & 0.756 & 0.008 & -293.790 \\ -0.068 & -0.084 & 0.977 & 126.250 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to ensure accuracy and reliability of calibration results, the experimental robot we choose has higher positioning accuracy and meet algorithm demand. Then we obtain the hand-eye results by solving many times and the least squares fitting to reduce error and uncertainty.

This article takes the hand-eye matrix  $X$  into traditional equation  $AX=XB$ , through comparing  $AX$  and  $XB$  values to verify the accuracy of the algorithm.  $A$  represents the camera after moving pose relative to the previous one.  $B$  represents the manipulator after moving pose relative to the previous one. Finally, we make the robot moving three times and get two groups  $A$  and  $B$  values, error  $err=AX-XB$ , which are shown in table 1.

**Table 1 The hand-eye matrix error**

$AX$	$XB$	$err$
$\begin{bmatrix} 0.717 & 0.546 & 0.119 & -42.754 \\ -0.692 & 0.834 & 0.011 & -322.463 \\ -0.079 & -0.077 & 0.977 & 127.315 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.682 & 0.650 & 0.117 & -43.336 \\ -0.780 & 0.756 & 0.008 & -323.758 \\ -0.068 & -0.084 & 0.977 & 128.692 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.035 & -0.104 & 0.002 & 0.582 \\ 0.088 & 0.078 & 0.003 & 1.295 \\ -0.011 & 0.007 & 0 & -1.377 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$\begin{bmatrix} 0.616 & 0.655 & 0.122 & -81.554 \\ -0.784 & 0.751 & 0.009 & -332.361 \\ -0.069 & -0.089 & 0.976 & 131.021 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.622 & 0.650 & 0.117 & -79.681 \\ -0.780 & 0.756 & 0.008 & -331.234 \\ -0.068 & -0.084 & 0.977 & 131.493 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -0.006 & 0.005 & -0.05 & -1.873 \\ -0.04 & -0.005 & 0.001 & -1.037 \\ -0.001 & -0.005 & -0.001 & -0.472 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

According to the data in table 1, the rotation matrix error up to 0.104mm, the translation matrix error up to 1.873mm. Compared with traditional calibration method, the proposed method improves the accuracy and proves that the hybrid method has sufficient accuracy and effectiveness, and can be used in practice.

## CONCLUSION

This article put forward a hand-eye hybrid calibration method by combining the active vision and traditional vision. The hand-eye rotation matrix  $R$  is obtained by making twice robotic precise translational motion (general orthogonal movement) and photograph calibration board respectively. The translation matrix  $T$  can be solved by controlling the robot to move to an image point location and combining with  $R$ . The algorithm achievement steps and experimental process of the proposed method are given in this paper, then we discuss how to reduce error, and finally prove the accuracy of the calibration results. This method avoids the traditional complicated equations and doesn't need the camera perform elusive pure rotary motion and the calibration template is simple to make and has high precision. In short, the proposed method is easy to implement and can be powerful generalization.

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